

Theoretical and experimental study of a 300-W beta-type Stirling engine



Chin-Hsiang Cheng*, Hang-Suin Yang, Lam Keong

Institute of Aeronautics and Astronautics, National Cheng Kung University, No. 1, Ta-Shieh Road, Tainan 70101, Taiwan, ROC

ARTICLE INFO

Article history:

Received 6 March 2013

Received in revised form

24 June 2013

Accepted 26 June 2013

Available online 31 July 2013

Keywords:

Stirling engine

Theoretical model

Experiment

Power output

ABSTRACT

In this study, a beta-type 300-W Stirling engine is developed and tested, and a non-ideal adiabatic model is built and applied to predict performance of the engine. Engine torque, engine speed and shaft power output are measured under various operating conditions. The experiments are conducted for two different working gases (air and helium) and at various charged pressures and heating temperatures. Effects of regenerator wire mesh on the shaft power output are also examined. Results show that the shaft power output of the engine is much higher using helium as the working fluid than using air. Furthermore, as the charged pressure and the heating temperature are set at 8 bars and 850 °C and a No. 120 wire mesh is used in the regenerator, the shaft power of the engine can reach 390 W at 1400 rpm with 1.21-kW input heat transfer rate (32.2% thermal efficiency). The experimental data are compared with the numerical predictions to verify the theoretical model. It is found that the experimental data of the shaft power output closely agree with the numerical predictions. This implies that the theoretical model is valid and helpful in the engine design.

© 2013 Elsevier Ltd. All rights reserved.

1. Introduction

Stirling engines have been used in CSP (concentrating solar power) systems [1] that adopt mirrors or lenses to concentrate a large area of sunlight or solar thermal energy onto a small area. Electrical power is produced when the concentrated sunlight is converted to thermal energy to drive the Stirling engine and connected to an electrical power generator. In recent year, CSP systems are also used to combine with fuel cells for stable power source [2]. The electrical power generated by CSP systems is connected to electrolyser, produce hydrogen and store. Then use fuel cell to supply electrical power even in night. Since Stirling engine is referred to as an external combustion engine, in addition to CSP systems, it is also suitable for variable external heat sources, including waste heat, geothermal energy and combustion heat from fossil fuels or hydrogen provided that heating temperature of any external heat source is sufficiently high. For example, Cullen and McGovern [3] investigate the feasibility of utilizing a Stirling cycle engine as an exhaust gas waste heat recovery device for an internal combustion engine. In addition, Stirling engine is also used in a combined heat and power system [4,5].

* Corresponding author. Tel.: +886 6 2757575x63627; fax: +886 6 2389940.
E-mail address: chcheng@mail.ncku.edu.tw (C.-H. Cheng).

The first patent of the Stirling engine was filed by Robert Stirling in 1817 [6] who originally intended to invent a hot-air closed-cycle prime mover to serve as an alternative to the explosively dangerous steam engine. In 1871, Schmidt [7,8] proposed a simple second-order analysis of the ideal Stirling engine cycle. In order to modify the isothermal models, Finkelstein [9] and Urieli, Rallis and Berchowitz [10] presented ideal adiabatic models, in which the thermodynamic processes taking place in the expansion and the compression spaces are assumed to be adiabatic processes. Recently, Cheng and Yu [11] took into consideration the temperature difference between the working gas and the external heat source, pressure drop in the regenerator, and effectiveness of the regenerator and developed a more realistic model for the beta-type Stirling engine. Based on the development of adiabatic model, Parlak et al. [12] considered dissipation and convection losses and built a non-ideal adiabatic model to analyze gamma-type Stirling engine. Timoumi et al. [13] take the losses due to conduction, shuttle effect, and gas spring hysteresis into consider, and optimize GPU-3 Stirling engine. More recently, Yu et al. [14] developed a non-ideal adiabatic model for a double-acting Stirling engine by introducing effectiveness of regenerator, conductive heat loss and pressure drop. Senft [15] proposed the concept of forced work as well as mechanism effectiveness for determining the shaft work of the engine. Lately, Makhkamov and Ingham [16] evaluated the friction loss due to sealing rings and bearings, and then obtained

Nomenclature

A	total heat transfer area (m^2)
A_d, A_p	cross section area of displacer and piston (m^2)
c_p, c_v	isobaric and isochoric specific heat (J/kg K)
d_p	bore size of piston (m)
$d_{h,r}$	hydraulic diameter of mesh (m)
d_w	diameter of mesh wire (m)
E	the effectiveness of mechanism
f_r	Fanning friction factor of regenerator
h	heat transfer coefficient ($\text{W/m}^2 \text{K}$)
l_1, l_2, l_3, l_4	length of the links (m)
l_b, l_d, l_p	length of the displacer bar, displacer, and piston (m)
l_r	length of regenerator (m)
l_t	the distance from the center of shaft to the top of cylinder (m)
m	mass of working fluid (kg)
$\dot{m}$	mass flow rate (kg/s)
NTU	number of transfer unit
Nu	Nusselt number
p	pressure (N/m^2)
Q	heat (J)
Q_{loss}	heat loss due to imperfect regeneration
r	offset distance from the crank to the center of shaft (m)
R	gas constant (J/kg K)
R	radius of gear (m)
Re	Reynolds number
s	porosity of wire mesh
t	time (s)

T	temperature (K)
u	average flow velocity (m/s)
V	volume (m^3)
V_{sw}	displacer swept volume (m^3)
W_i	indicated work (J)
W_{loss}	work loss due to pressure drop (J)
W_s	shaft work (J)
$\dot{W}_s$	shaft power (W)
W_-	forced work (J)
y_p, y_d	positions of piston and displacer (m)

Greek symbols

ε	effectiveness of regenerator
φ	crank angle (rad)
γ	ratio of specific heats, c_p/c_v
η_{th}	thermal efficiency
λ	mesh number ($1/\text{in.}$)
ρ	density of working gas (kg/m^3)
τ	torque (Nm)
ω	rotation speed of engine (rpm)

Subscripts

c	compression chamber
e	expansion chamber
h	heater
k	cooler
r	regenerator
w	wall

the work loss. Moreover, Cheng and Yang [17] included the effects of imperfect heat transfer and regeneration to evaluate the ratio of temperatures of the heating to the cooling spaces, and adopted Senft's shaft work theory [15] to obtain the shaft power output as a function of engine speed. Cheng and Yu [18,19] extended the thermodynamic analysis to a dynamic simulation of a beta-type Stirling engine by combining the thermodynamic model presented in Ref. [11] and a dynamic model. The dynamic behavior of the engine in the starting period and dependence of the shaft power of the engine on the engine speed could then be predicted.

On the other hand, development of prototype engines is also in progress. Among all scales, the domestic-scale or personal Stirling electric power generators of roughly several hundred Watts are of great market potential. The Stirling engine of this scale features light weight and low maintenance cost, and hence, it is particularly suitable for applications as a mobile personal power generator. For example, Cinar and Karabulut [20] manufactured and tested a gamma-type Stirling engine using different working fluids under different charged pressures. Authors obtained a maximum power of 128.3 W with their engine. Lately, Karabulut and coworkers [21,22] developed a beta-type Stirling engine using helium and measured the torque and the power produced by the engine. They studied the influence of surface treatment on the performance of the engine and obtained a maximum power of 250 W at rotation speed of 545 rpm. Recently, Sripakagorn and Srikam [23] designed a beta-type Stirling engine using air and measured the shaft power of it. With their engine, authors obtained a maximum power of 95.4 W at 360 rpm. The basic data and engine specifications are summarized in Table 2.

In general, in terms of configuration, the Stirling engines can be divided into three major categories, namely, alpha-, beta-, and gamma-types. Detailed description regarding these three categories

is available in Ref. [24]. Among these three categories, the beta-type engine has higher power density than the other two types, as shown by Cheng and Yang [25] and Kirkley [26]. Therefore, it is chosen in this study to make a domestic-scale engine of 300 W. In this study, the beta-type Stirling engine with rhombic-drive mechanism [27] is developed. With the help of Senft's theory [15] the shaft power output can be determined numerically.

2. Prototype engine

The photo of the prototype engine is shown in Fig. 1, and the schematic of the beta-type Stirling engine with rhombic drive

Table 1
Design parameters of prototype engine.

Parameter	Value
$l_1 = l_2(\text{m})$	0.015
$l_3 = l_4(\text{m})$	0.04
$r(\text{m})$	0.016
$R(\text{m})$	0.03
$d_p(\text{m})$	0.07
$l_p(\text{m})$	0.083
$l_b(\text{m})$	0.164
$l_d(\text{m})$	0.087
$l_t(\text{m})$	0.233
Working fluid	Air or Helium
Charged pressure	4–8 bar
Stroke	35 mm
Displacer swept volume	135 c.c.
Swept volume ratio	1
Dead volume ratio	1.13
Compression ratio	1.57
Heater type	Tubular heater
Cooler type	Fined water jacket

Table 2
Engine specifications in Refs. [20–23].

	Cinar C. et al. [20]	Karabulut H. et al. [21]	Karabulut H. et al. [22]	Sripakagorn A. et al. [23]
Engine type	γ	β	β	β
Displacer swept volume	138 c.c.	295 c.c.	295 c.c.	165 c.c.
Swept volume ratio	1	0.78	0.78	1
Working fluid	Helium and air	Helium	Helium	Air
Charged pressure	1–4 bar	2–4.8 bar	1–4.5 bar	1–7 bar
Heating temperature	800–1000 °C	360 °C	180–260 °C	350–500 °C
Cooling system	Water cooled	Water cooled	Water cooled	Water cooled
Compression ratio	1.82	1.72	1.65	1.61
Maximum power and engine speed	128.3 W at 891 rpm	250 W at 545 rpm	183 W at 590 rpm	95.4 W at 360 rpm

mechanism is shown in Fig. 2. The rhombic drive, which is frequently used with the beta-type Stirling engines [19], utilizes a jointed rhomboid to convert linear motion of a reciprocating piston to a rotational of flywheel. In the rhombic drive mechanism, one rigid rod is installed connecting the piston to the top corner of the jointed rhomboid, and another rigid rod connecting the displacer to the bottom corner of the rhomboid. In addition, two symmetric gears of equal diameter are connected to the right and the left corners of the rhomboid fixed on the gears at an offset distance from gears center. The two gears are in contact and rotate in opposite directions. When gas pressure is applied to the piston, the top corner of the rhomboid is pushed downward; the rhomboid is flattened in the direction of the piston axis; and then it pushes on the gear wheels and causes them to rotate. At the same time, as the gear wheels rotate, the rhomboid progresses its change of shape and drives its bottom corner and the displacer to move upward. The design parameters of the prototype engine are tabulated in Table 1. Here in this table, Swept volume ratio is defined as the ratio of the volume swept by piston to the volume swept by displacer. And, dead volume ratio is the ratio of dead volume to the volume swept by displacer.

In order to increase total heat transfer area and to reduce thermal resistance, the hot part of the engine is designed to consist

of 12 stainless tubes. On the other hand, the cold part is cooled by water cooling with a finned water jacket. The material of the cold part is made of aluminum alloy. The displacer and the piston are both made of light-weight aluminum alloy to reduce reciprocating inertial moment and vibration. The prototype engine is not a hermetic engine since the flywheel is extruded through the crankcase. In order to prevent gas leaking, the contact surfaces of the flywheel shaft and the crankcase should be polished and a high-precision mechanical gas seal is equipped at the interface. The charge pressure can be maintained as the seal is used.

In addition, a regenerator made of stainless wire meshes is installed in between the hot and the cold parts. The properties of the stainless wire meshes are essential to the friction loss and the effectiveness of the regenerator. For investigating the effects of the stainless wire mesh on the performance of the engine, the stainless wire mesh of number 50, 100, 120, 150, and 165 are tested. The mesh number is already standardized and is a measure of fineness of the mesh. The numerical value indicates the number of openings per linear inch. The smaller the mesh number, the larger is the opening area.

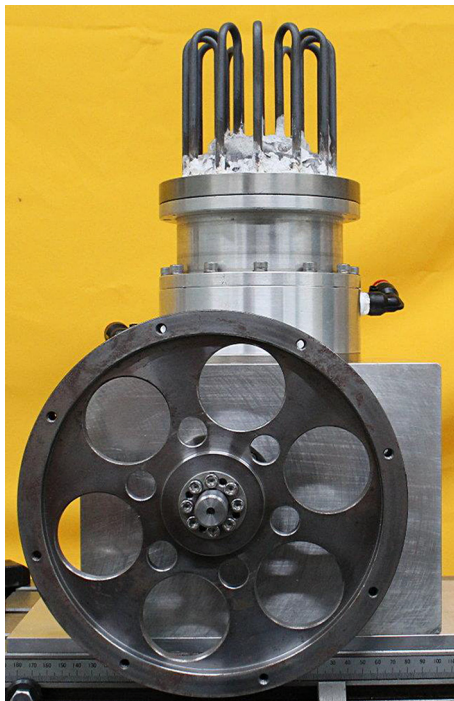


Fig. 1. Photographic copy of prototype engine.

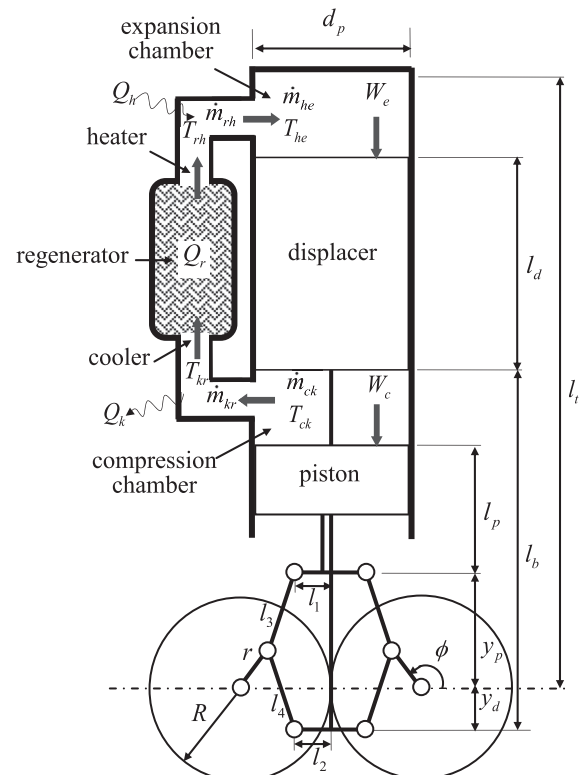


Fig. 2. Schematic diagram of beta-type Stirling engine with rhombic drive mechanism.

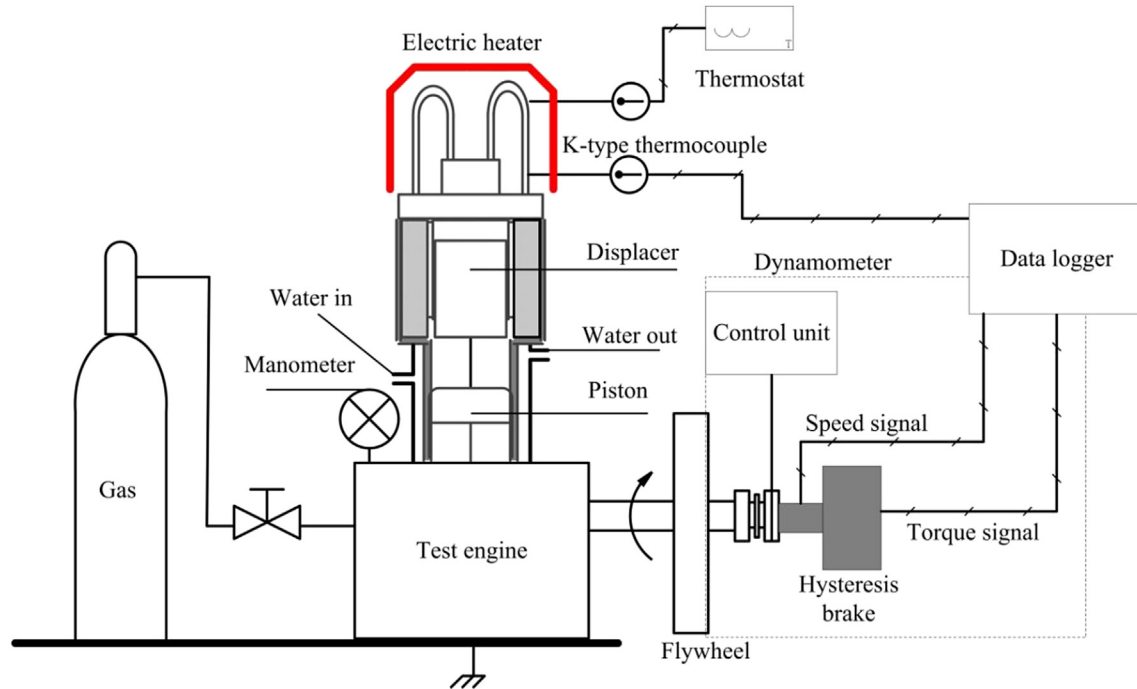


Fig. 3. Experimental apparatus and measuring devices.

The experimental apparatus for the engine performance test is shown in Fig. 3. An electric infrared heater, FIBROTHAL, of 1.21 kW (11 A/110 V) is used as the external heat source, and a thermostat, TAIE FY800, is used to control the heating temperature. The heating temperature is up to 1000 °C and with ± 1 °C of accuracy at steady state. A K-type thermocouple with $\pm 0.75\%$ of accuracy is placed in the center of the electric infrared heater to monitor the heating temperature and the temperature data are recorded by a FLUKE data logger. In experiments, the engine load is applied on the crank shaft of the engine by using a hysteresis brake dynamometer, MAGTROL, and the brake torque is exerted by dynamometer controller. The dynamometer provides brake torque loading independent of the engine speed. The rotation speed of the engine and the brake torque can be measured by a rotation meter and a torque sensor, respectively. The measurement range is 0–5000 rpm and ± 5 Nm. The data are also recorded by the data logger. The shaft power is then determined in terms of the brake torque and the engine rotation speed. The input heat transfer rate from the electric infrared heater is fixed at 1.21 kW. The ratio of the shaft power output to the input heat transfer rate represents the thermal efficiency of the engine.

Uncertainty analysis is performed and results are tabulated in Table 3. The sources of uncertainties may be caused by measuring system errors, system–sensor interaction errors, system disturbance errors, and conceptual errors. The method of uncertainty

analysis presented by Moffat [28] is used in this study. The uncertainty of each measured variables like heating temperature $T_{h,w}$, torque τ , and engine speed ω are estimated with 95% confidence. Table 3 shows that the interval of heating temperature is between 650 and 850 °C with largest relative uncertainty 2.35%. The relative uncertainties of torque and engine speed are lower than 0.3%. The relative uncertainty of shaft power $\dot{W}_s$ is 0.35%, which calculated by using root sum-square method proposed in Ref. [28].

3. Theoretical model

In this study, a non-ideal adiabatic model that involves the friction power loss due to pressure drop and the heat loss due to imperfect regeneration is proposed. Meanwhile, Senft's theory [15] is also included to estimate the shaft power output and the effectiveness of the mechanism. The theoretical model is briefly described as follows:

The assumptions with the model are:

- (1) The pressure is uniform throughout the interior space of the engine.
- (2) The expansion chamber and the compression chamber are insulated.
- (3) The temperature distribution in the regenerator is varied linearly from the heater's temperature to the cooler's temperature.
- (4) The working gas is regarded as an ideal gas.

For the mechanism illustrated in Fig. 2, the displacements of the piston and the displacer can be calculated, respectively, by

$$y_p = r \sin \varphi + \sqrt{l_3^2 - (R - l_1 + r \cos \varphi)^2} \quad (1a)$$

$$y_d = r \sin \varphi - \sqrt{l_4^2 - (R - l_2 + r \cos \varphi)^2} \quad (1b)$$

Then, the volumes variation of the expansion chamber and the compression chamber are determined by

Table 3
Uncertainty analysis for measured variables.

Variable	Typical value	Uncertainty ^a	Relative uncertainty
	x	δx	$\delta x/x$
$T_{h,w}$	650–850 °C	20 °C	2.35%
τ	0–3.5 Nm	0.01 Nm	0.29%
ω	0–2500 rpm	5 rpm	0.20%
$\dot{W}_s$	0–400 W	—	0.35% ^b

^a All estimated with 95 percent confidence.

^b $\delta \dot{W}_s / \dot{W}_s = [(\delta \tau / \tau)^2 + (\delta \omega / \omega)^2]^{1/2}$.

$$V_e = A_d(l_t - l_d - l_b - y_d) \quad (2a)$$

$$V_c = A_p(y_d + l_b - l_p - y_p) \quad (2b)$$

where $A_d = A_p = \pi d_p/4$, denoting the transverse area of the displacer and the piston.

Since the pressure is assumed to be uniform throughout the interior space of the engine, the law of mass conservation leads to

$$p = mR / \left(\frac{V_c}{T_c} + \frac{V_k}{T_k} + \frac{V_r}{T_r} + \frac{V_h}{T_h} + \frac{V_e}{T_e} \right) \quad (3)$$

With the help of ideal-gas equation of state, the mass variation due to the pressure variation in the cooler, the regenerator, and the heater are determined as

$$dm_i = V_i dp / RT_i \quad \text{where the subscript } i \equiv k, r, h \quad (4)$$

Meanwhile, the mass variation in the expansion chamber and the compression chamber can be determined as

$$dm_e = (pdV_e + V_e dp / \gamma) / RT_{he} \quad (5a)$$

$$dm_c = (pdV_c + V_c dp / \gamma) / RT_{ck} \quad (5b)$$

In Eq. (5b), T_{ck} is the temperature of the working gas flowing through the boundary between the compression chamber and the cooler, and is dependent on the flow direction. If $\dot{m}_{ck} \geq 0$, $T_{ck} = T_c$; If $\dot{m}_{ck} < 0$, $T_{ck} = T_k$. Similarly, T_{he} is the temperature of the working gas flowing through the boundary between the expansion chamber and the heater. If $\dot{m}_{he} \geq 0$, $T_{he} = T_h$; If $\dot{m}_{he} < 0$, $T_{he} = T_e$. Thus, the mass flow rate between any two chambers can be calculated by

$$\dot{m}_{he} = \frac{dm_e}{dt} \quad (6a)$$

$$\dot{m}_{ck} = -\frac{dm_c}{dt} \quad (6b)$$

$$\dot{m}_{kr} = \dot{m}_{ck} - \frac{dm_k}{dt} \quad (6c)$$

$$\dot{m}_{rh} = \dot{m}_{he} + \frac{dm_h}{dt} \quad (6d)$$

Since total mass of the working gas is fixed, one obtains the pressure variation from Eq. (3) as

$$dp = \frac{-\gamma p (dV_c / T_{ck} + dV_e / T_{he})}{V_c / T_{ck} + \gamma (V_k / T_k + V_r / T_r + V_h / T_h) + V_e / T_{he}} \quad (7)$$

The temperatures in the expansion chamber and the compression chamber can be calculated by using equation of state as below:

$$dT_i = T_i \left(\frac{dp}{p} + \frac{dV_i}{V_i} - \frac{dm_i}{m_i} \right) \quad \text{where the subscript } i \equiv e, c \quad (8)$$

Note that the temperatures in the expansion and the compression chambers are not constant but varied as periodic functions of time.

Initially, one needs to specify the rotation speed of the engine (ω). Then, at every time step, the numerical solutions for volumes, pressure, temperatures, a masses in different parts of the engine are determined in iteration using Eqs. (2)–(8). Once the convergence of iteration is achieved, heat transfers during the time step in the

cooler, the regenerator, and the heater can be obtained based on the first law of thermodynamic as

$$\delta Q_k = c_v V_k dp / R + (\dot{m}_{kr} T_{kr} - \dot{m}_{ck} T_{ck}) c_p dt \quad (9a)$$

$$\delta Q_r = c_v V_r dp / R + (\dot{m}_{rh} T_{rh} - \dot{m}_{kr} T_{kr}) c_p dt \quad (9b)$$

$$\delta Q_h = c_v V_h dp / R + (\dot{m}_{he} T_{he} - \dot{m}_{rh} T_{rh}) c_p dt \quad (9c)$$

The net indicated work, heat output in the cooler, and heat input in the heater per cycle are then calculated with

$$W_i = \oint p dV_e + \oint p dV_c \quad (10a)$$

$$Q_k = \oint \delta Q_k \quad (10b)$$

$$Q_h = \oint \delta Q_h \quad (10c)$$

respectively.

Heat rejected in the cooler and absorbed in the heater are expressed by

$$Q_k = 60 h_k A_k (T_{k,w} - T_k) / \omega \quad (11a)$$

$$Q_h = 60 h_h A_h (T_{h,w} - T_h) / \omega \quad (11b)$$

Eq. (11) are used to determine T_h and T_k once the heat rejected in the cooler and heat absorbed in the heater are obtained from Eq. (9). In the above equation, ω is the engine rotation speed and $T_{h,w}$ and $T_{k,w}$ are the wall temperatures in the heater and the cooler, respectively. Heat transfer coefficients on the surfaces of the heater and the cooler (h_h and h_k) are evaluated by the empirical correlation between Nusselt and Reynolds numbers presented in Ref. [29], which reads

$$Nu = a Re^b \quad (12)$$

where Nu and Re are defined in terms of sizes of the heater and the cooler and average velocities in the chambers, which is evaluated from the data of mass flow rates. The coefficients a and b are obtained from experiments. In this study, a and b are chosen to be 0.035 and 0.8, respectively.

Next, work loss due to the pressure drop through the regenerator, that was not included in a uniform-pressure model, need to be considered. As illustrated in Refs. [30,31], through a real regenerator the pressure drop can be expressed as

$$\Delta p_r = f_r (l_r / d_{h,r}) \rho u^2 / 2 \quad (13)$$

where l_r is the length of regenerator; ρ is the density of working gas; u is the gas ass flow rate which can be calculated from Eqs. (6a)–(6d); and f_r is the friction factor which can be calculated from the empirical correlations in Ref. [31]. In addition, $d_{h,r}$ is the hydraulic diameter of wire mesh which is defined in terms of the mesh number λ and the diameter of wire d_w as in Ref. [30].

$$d_{h,r} = d_w s (1 - s) \quad (14)$$

where d_w denotes the diameter of mesh wire and s represents the porosity of the mesh which is determined by $s = 1 - (\lambda \pi d_w) / 4$.

As discussed in Ref. [14], the work loss due to the pressure drop through the regenerator is evaluated by

$$W_{\text{loss}} = \oint \Delta p_r dV_e \quad (15)$$

On the other hand, heat loss due to the imperfect regeneration can be calculated in terms of the effectiveness of regenerator as

$$Q_{\text{loss}} = Q_r(1 - \varepsilon) \quad (16)$$

where the effectiveness of regenerator (ε) is expressed as a function of the NTU (number of transfer unit) of the regenerator. That is

$$\varepsilon = \text{NTU} / (1 + \text{NTU}) \quad (17)$$

with $\text{NTU} = A_r \bar{h}_r / c_p \bar{m}_r$; A_r is total heat transfer area of wire mesh in regenerator; and $\bar{h}_r$ is average heat transfer coefficient in the regenerator which can be obtained by using the empirical correlations provided in Ref. [30].

As long as the p – V diagram of the thermodynamic cycle is obtained, one may evaluate the forced work per cycle W_- , the indicated work per cycle W_i , and the work loss per cycle W_{loss} . Based on Senft's theory [15], the shaft work of engine per cycle W_s can be determined in terms of the indicated work and the forced work as

$$W_s = E(W_i - W_{\text{loss}}) - (1/E - E)W_- \quad (18)$$

where E is the effectiveness of mechanism. The mechanism effectiveness is well known as a ratio measure of how well a mechanism utilizes work if it is regarded as a work transmitter. As discussed in Refs. [15,25], it is a ratio of the work it transfers out through an actuator or puts into internal storage to the work it receives through another actuator or takes from its internal source. Typical value of the mechanical effectiveness is in the range of $0.7 < E < 0.9$. In this study, the mechanism effectiveness is adjusted to be 0.82 by curve-fitting with the experimental data. Detailed information for the mechanism effectiveness has been provided by Senft [15] and Cheng and yang [25]; therefore, no further description is provided here to save space. Finally, the shaft power of the engine ($\dot{W}_s$) is the product of the shaft work per cycle (J) and the engine rotation speed (cycles per second). Similarly, the torque exerts on the shaft can be calculated by using Eq. (18). Assume the inertial of flywheel is large enough that the engine rotation speed fluctuation over a cycle is significant small, the shaft power can be calculated by torque (Nm) times engine rotation speed (rad/s) approximately. As has been discussed earlier, shaft power is equal to the product of the shaft work per cycle and the engine rotation speed. Thus, the engine torque can be calculated by

$$\tau = \frac{W_s}{2\pi} \quad (19)$$

Furthermore, the relation of thermal efficiency can be expressed as

$$\eta_{\text{th}} = \frac{W_s}{Q_h + Q_{\text{loss}}} \quad (20)$$

4. Results and discussion

In this section, effects of the mesh number of the regenerator, the charged pressure, the heating temperature are evaluated experimentally and theoretically. The difference in the performance of engines using air and helium as the working gas is also observed.

Fig. 4 shows the experimental data of the shaft power as a function of the engine rotation speed and the mesh number. In the cases, air at 4 or 6 bars is used as the working gas and the heating temperature is fixed at 650 °C. According to the experimental measurement, it can be found that the shaft power reaches a maximum at a critical engine speed. As the engine speed is lower or higher than the critical speed, the shaft power is decreased. This probably is attributed to the fact that when the engine speed is increased, the pressure loss is elevated and the gas temperature difference between the expansion and the compression chambers is decreased due to the imperfect heat transfer [17]. On the contrary, when the engine speed is decreased, the magnitude of the shaft power tend to decrease with the engine speed since the shaft power is actually the product of the shaft work per cycle and the engine rotation speed. This explains why a critical engine speed is

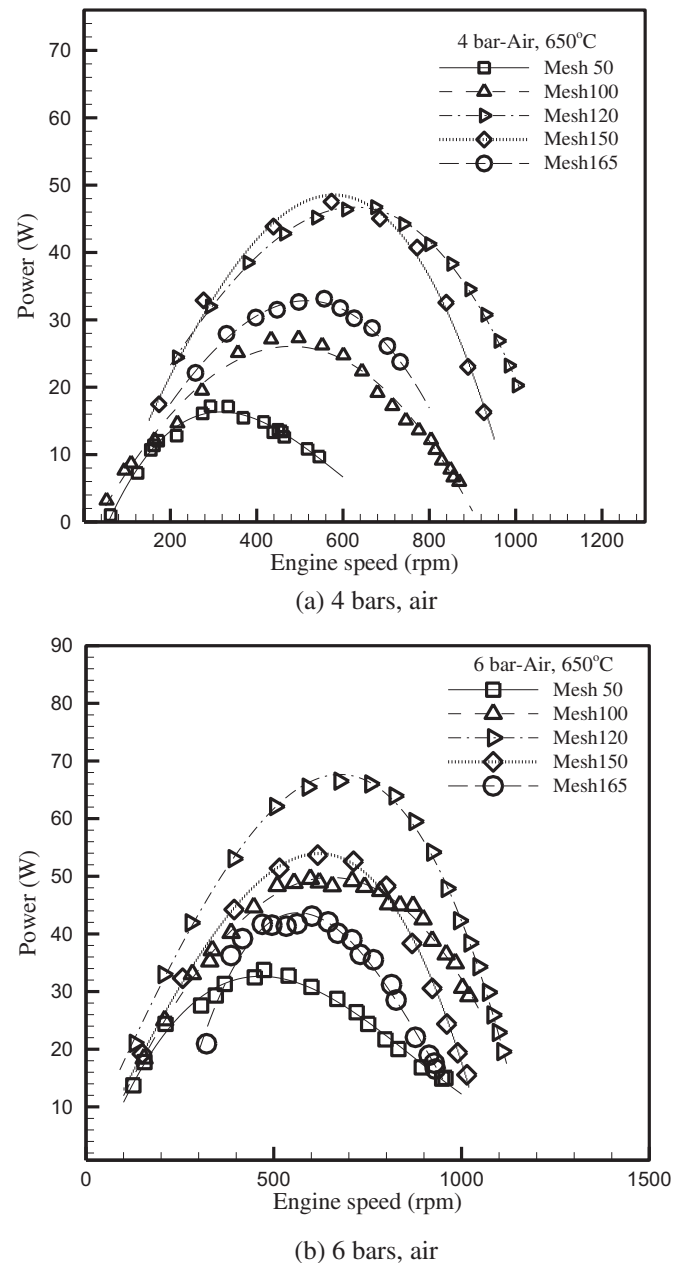
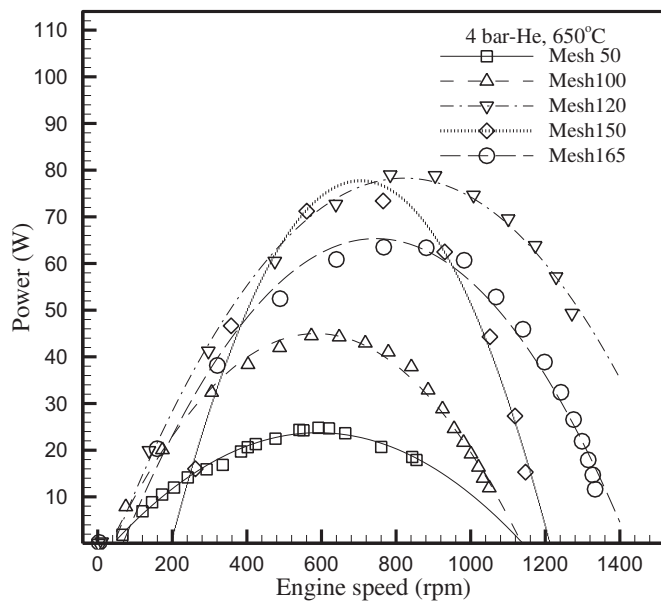
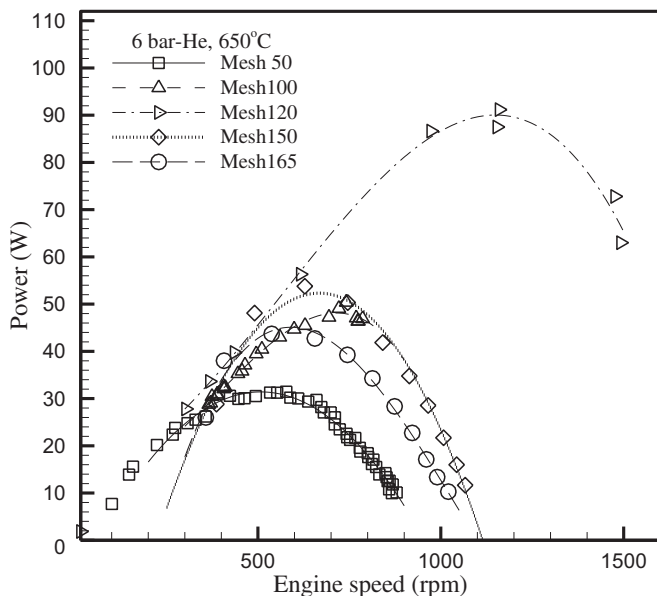


Fig. 4. Experimental data of shaft power versus engine speed and mesh number. Air at 4 or 6 bars is used and heating temperature is fixed at 650 °C. (a) 4 bars, air, (b) 6 bars, air.

found. Meanwhile, it is noted that the critical engine speed is varied when the mesh number of the regenerator is changed. As the mesh number is increased, both the pressure drop through the regenerator and the heat transfer area in the regenerator increase with the mesh number. Therefore, due to a balance between the positive and the negative effects caused by an increase in the mesh number, there should be an optimal mesh number expected. The results shown in the plots of Fig. 4 reveal that the optimal mesh number is 120, which leads to highest shaft power output. Mesh number 150 produces highest shaft power only in the low-speed regime at 4 bars. Using air at 650 °C, the engine produces maximum shaft power of 47 W at 4 bars and 60 W at 6 bars.



(a) 4 bars, helium



(b) 6 bars, helium

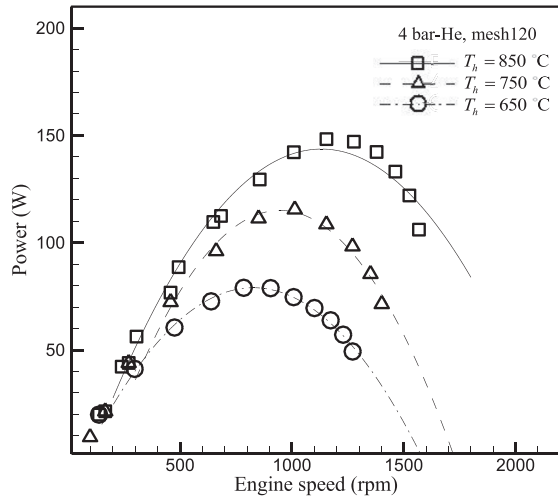
Fig. 5. Experimental data of shaft power versus engine speed and mesh number. Helium at 4 or 6 bars is used and heating temperature is fixed at 650 °C. (a) 4 bars, helium, (b) 6 bars, helium.

Fig. 5 shows the experimental results with helium for the same heating temperature (650 °C), charged pressures and mesh numbers. Firstly, it is observed that when helium is charged into the engine as the working gas, the experimental data exhibits a great increase in the shaft power. Using helium at 650 °C and with mesh 120, the engine produces maximum shaft power of 80 W at 4 bars and 92 W at 6 bars. It is also observed that as helium is in use, the critical engine speed is increased. The reason for the improvement in engine performance with helium is that helium possesses higher thermal diffusivity and lower viscosity than air. However, a comparison between Fig. 5(a) and (b) illustrates that as the helium is in use, the shaft power output may still be decreased even though the pressure is increased. In Fig. 5(b), one observes a remarkable increase in the power output when using mesh number 120. Engine performance is strongly dependent on the heat transfer effectiveness and friction loss in the device, which are influenced by the properties of the porous medium used in the regenerator. In general, as the mesh number increases, the porosity of the porous medium is decreased and the heat transfer area is increased. As a result, the friction loss and the heat transfer rate both are elevated. An increase in the friction loss reduces the engine performance, while an increase in heat transfer rate elevates the engine performance. The opposite effects of the mesh number on the performance of engine imply that there exists an optimal mesh number with the regenerator. For the particular prototype engine developed here, mesh number 120 leads to an optimal engine performance according to the experiments. This seemingly implies that for the cases at higher pressure the mesh number is rather influential. A higher pressure means higher density. The work loss accompanying the pressure drop through the regenerator increases with the density of the working gas. Therefore, in order to improve the performance of Stirling engine at high charged pressure, heat transfer must be elevated by increasing the heating temperature or the heat transfer area. If the heat transfer in the regenerator is not sufficient, an increase in charged pressure may cause a reduction in the shaft power.

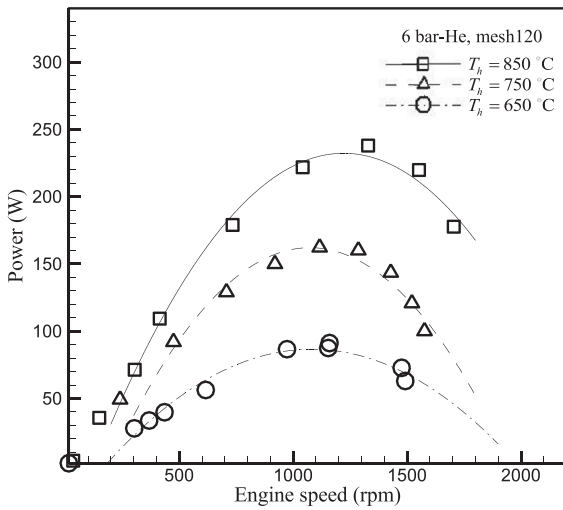
In accordance with the above discussion, further study is focused on the effects of heating temperature on the performance of the high-pressure, helium-charged engine. Fig. 6 shows the experimental data of the dependence of the shaft power on the heating temperature. In this figure, the helium-charged engine at 4, 6, or 8 bars is tested. The heating temperatures of 650, 750, and 850 °C are considered. It is noticed that the shaft power output and the engine speed appreciably increase with the heating temperature. For the case with wire mesh of number 120, helium at 8 bar and heating temperature of 850 °C, the engine is able to produce a 390-W shaft power output at about 1400 rpm. In this case, the corresponding thermal efficiency of the engine is 32.2%.

Fig. 7 conveys the comparison between experimental and numerical data in the shaft power as a function of engine speed and mesh number. In the plots of this figure, the cases with 6-bar air and 650-°C heating temperature [Fig. 7(a)] and 8-bar helium and 850-°C heating temperature are presented. The numerical predictions of the shaft power are higher than the experimental data by 12% ~ 20% for all cases. The difference between numerical predictions and experimental data is probably attributed to the uncertainty of the experiments or the assumptions made in the theoretical model. It is not possible to clarify this at the present moment. Therefore, further study will be definitely required. Nevertheless, the agreement in the tendencies of both sets of data is observed. Based on the numerical predictions, it is also found that the regenerator with mesh number 120 leads to a maximum shaft power output. This exactly agrees with the experimental observation.

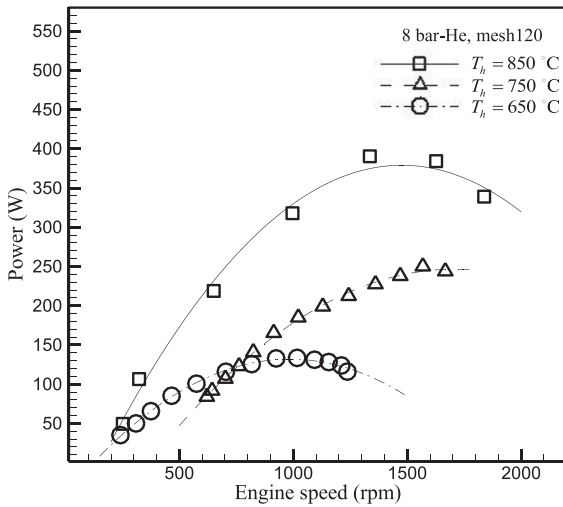
Comparison in torque of engine between experimental and numerical data is made, and the results are conveyed in Fig. 8. The



(a) 4 bars

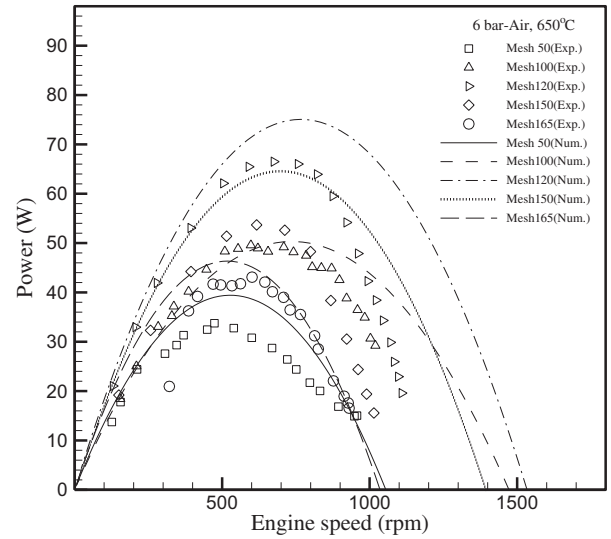


(b) 6 bars

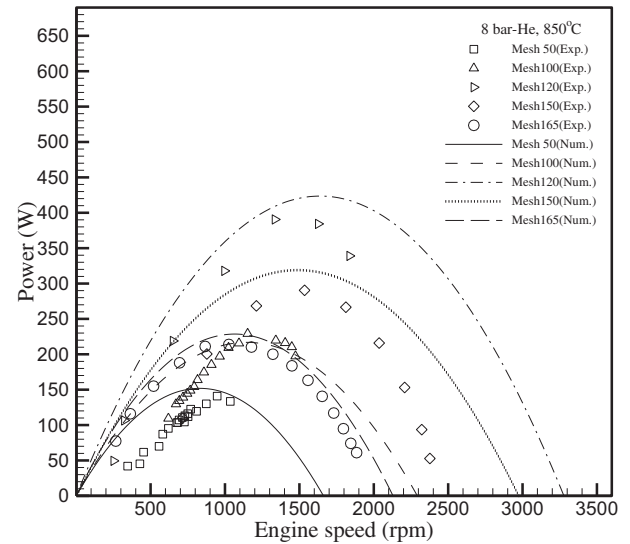


(c) 8 bars

Fig. 6. Experimental data showing dependence of shaft power on heating temperature. Helium at 4, 6, and 8 bars is used. (a) 4 bars, (b) 6 bars, (c) 8 bars.



(a) 6-bar air and 650-°C heating temperature



(b) 8-bar helium and 850-°C heating temperature

Fig. 7. Comparison between experimental and numerical data, for shaft power versus engine speed and mesh number. (a) 6-bar air and 650-°C heating temperature, (b) 8-bar helium and 850-°C heating temperature.

torque of the engine is equal to shaft work divided by engine speed. Therefore, once the shaft power in terms of shaft work per cycle and engine speed is obtained, the torque can be calculated. In this figure, the engine with 6-bar air and 650-°C heating temperature is tested, and the torque of the engine is regarded as a function of engine speed and mesh number. Based on the numerical predictions, it is observed that for all cases the torque of engine is decreased linearly with increasing engine speed. This may also be caused by the smaller gas temperature difference between the expansion and the compression chambers with a higher engine speed. The discrepancy between the numerical and the experimental data appears to grow at a higher engine speed. Nevertheless, the experimental data basically agree with the numerical predictions even though some discrepancy is still seen, and both set of data reveal that mesh number 120 produces the highest engine

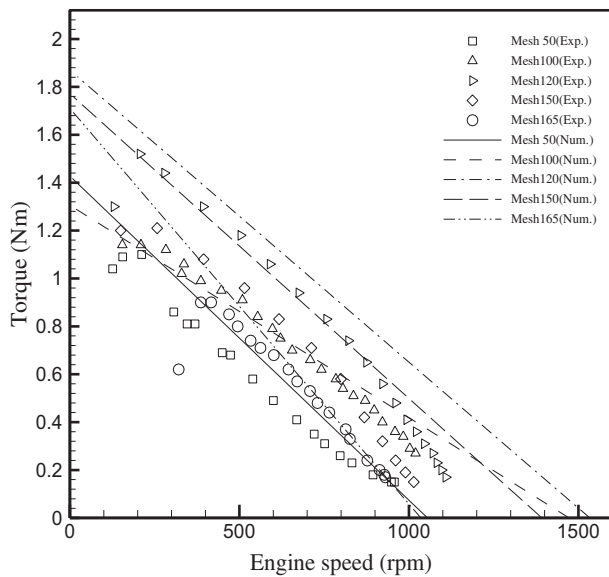


Fig. 8. Comparison between experimental and numerical data, for torque versus engine speed and mesh number, with 6-bar air and 650-°C heating temperature.

torque. For the cases shown in this figure, the engine torque is varied within 0.2–1.6 Nm. As the parameters are changed to 8-bar helium and 850-°C heating temperature, the engine torque can be elevated to 0.5–3.6 Nm.

Fig. 9 shows the numerical predictions of thermal efficiency as a function of engine speed, for the case using 8-bar helium, mesh number 120, and 850-°C heating temperature. It is expected that a smaller gas temperature difference between the expansion and the compression chambers at a higher engine speed leads to a lower thermal efficiency. The results provided in Fig. 9 reflect this expectation. In this figure, the thermal efficiency of the engine is slightly reduced from 38% to 35.7% as the engine speed is elevated from 600 to 2100 rpm. It is noticed that at 1400 rpm, the thermal efficiency is predicted to be 36.4%, which is close to the

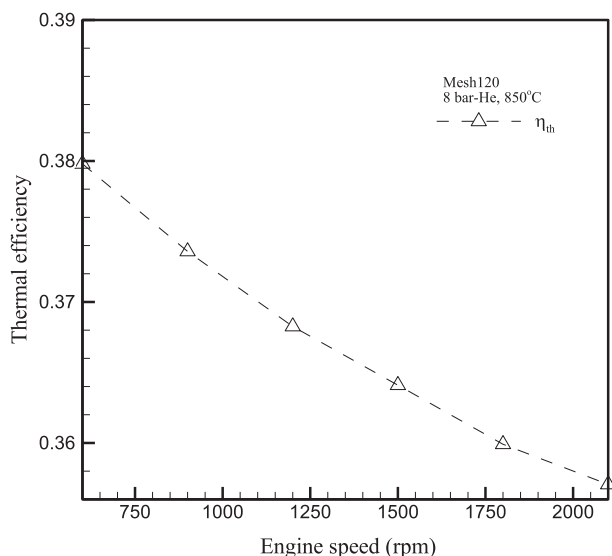


Fig. 9. Numerical predictions of thermal efficiency versus engine speed, for the case using 8-bar helium, mesh number 120, and 850-°C heating temperature.

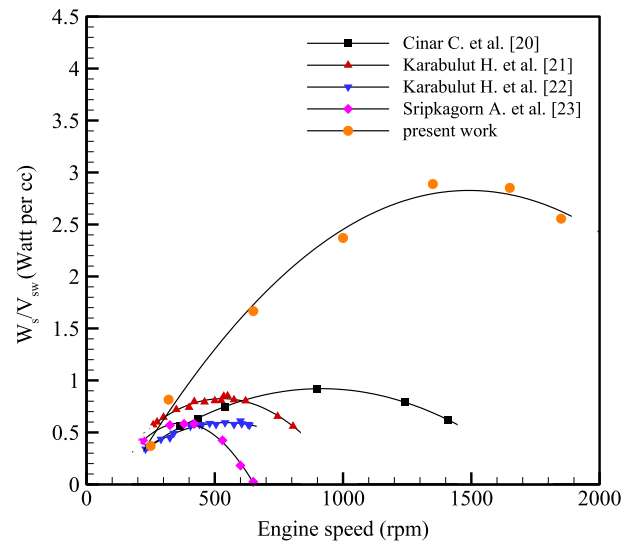


Fig. 10. Power density versus engine speed of the best case in Refs. [20–23] and present work.

experimental reading of 32.2%, which is already stated earlier. The validity of the present theoretical model is apparently ensured.

To compare the performance between the present and the existing similar engines, power density is defined as shaft power per displacer swept volume here. The curves of power density of different engines [20–23] are plotted as functions of engine rotation speed in Fig. 10. The corresponding engine specifications are listed in Table 2. It is found that the power density of present engine is relatively higher than the existing one. The power density of the present engine reaches 3.0 W/cc. High power density may be achieved by elevating the heating temperature or the charged pressure in the beta-type Stirling engines [25,26].

5. Conclusions

In this study, a domestic-scale 300-W Stirling engine with rhombic-drive mechanism is developed and tested. Experiments are conducted to test the performance of the engine under various operating conditions. Dependence of the shaft power output on the engine speed is investigated under different combinations of working gas, charged pressure, heating temperature, and regenerator wire mesh number. In parallel, a theoretical model is built and applied to predict the performance of the engine. With the help of Senft's theory [15] the shaft power output can be determined numerically.

Results show that the shaft power output of the engine is much higher using helium as the working fluid than using air. Furthermore, as the charged pressure and the heating temperature are set at 8 bars and 850 °C and a No. 120 wire mesh is used in the regenerator, the shaft power of the engine can reach 390 W at 1400 rpm with 1.21-kW input heat transfer rate, which means a thermal efficiency of 32.2%.

The experimental data are compared with the numerical predictions to verify the theoretical model. It is found that the numerical predictions of the shaft power are higher than the experimental data by 12% ~ 20%. Nevertheless, it is noticed that for the case using 8-bar helium, mesh number 120, and 850-°C heating temperature, the numerical prediction of the thermal efficiency is 36.4% at 1400 rpm. This value is close to the experimental reading of 32.2% at the same engine speed. The theoretical model is valid and helpful to the engine design.

Acknowledgment

This research was supported by the Southern Taiwan Science Park Administration (STSPA), Taiwan, R.O.C. under contract 101CP03.

References

- [1] Dracker R, De Laquil P. Progress commercializing solar-electric power systems. Palo Alto, CA, United States: Annual Reviews Inc; 1996. p. 371–402.
- [2] Petrescu S, Petre C, Costea M, Malancioiu O, Boriaru N, Dobrovicescu A, et al. A methodology of computation, design and optimization of solar stirling power plant using hydrogen/oxygen fuel cells. *Energy* 2010;35(2):729–39.
- [3] Cullen B, McGovern J. Energy system feasibility study of an Otto cycle/stirling cycle hybrid automotive engine. *Energy* 2010;35(2):1017–23.
- [4] Rogdakis ED, Antonakos GD, Koronaki IP. Thermodynamic analysis and experimental investigation of a Solo v161 stirling cogeneration unit. *Energy* 2012;45(1):503–11.
- [5] Parente A, Galletti C, Riccardi J, Schiavetti M, Tognotti L. Experimental and numerical investigation of a micro-CHP flameless unit. *Applied Energy* 2012;89(1):203–14.
- [6] Senft JR. Ringbom stirling engines. USA: Oxford University Press; 1993.
- [7] Schmidt G. Theorie der Lehmannschen calorischen maschine. *Zeitschrift des Vereines deutscher Ingenieure* 1871;15(1–12):97–112.
- [8] Thombare DG, Verma SK. Technological development in the stirling cycle engines. *Renewable and Sustainable Energy Reviews* 2008;12(1):1–38.
- [9] Finkelstein T. Computer analysis of stirling engines. *Advances in Cryogenic Engineering* 1975;20:269–82.
- [10] Urieli I, Rallis CJ, Berchowitz DM. Computer simulation of Stirling cycle machines. In: 12th intersociety energy conversion engineering conference: 1977 1977. p. 1512–21.
- [11] Cheng CH, Yu YJ. Numerical model for predicting thermodynamic cycle and thermal efficiency of a beta-type stirling engine with rhombic-drive mechanism. *Renewable Energy* 2010;35(11):2590–601.
- [12] Parlak N, Wagner A, Elsner M, Soyhan HS. Thermodynamic analysis of a gamma type stirling engine in non-ideal adiabatic conditions. *Renewable Energy* 2008;34(1):266–73.
- [13] Timoumi Y, Tlili I, Nasrallah SB. Design and performance optimization of GPU-3 stirling engines. *Energy* 2008;33(7):1100–14.
- [14] Yu Y, Yuan Z, Ma J, Zhu Q. Numerical simulation of a double-acting stirling engine in adiabatic conditions. *Advanced Materials Research* 2012;482:589–94.
- [15] Senft JR. Mechanical efficiency of kinematic heat engines. *Journal of the Franklin Institute* 1987;324(2):273–90.
- [16] Makhkamov KK, Ingham DB. Analysis of the working process and mechanical losses in a stirling engine for a solar power unit. *Journal of Solar Energy Engineering* 1999;121:121.
- [17] Cheng C-H, Yang H-S. Analytical model for predicting the effect of operating speed on shaft power output of stirling engines. *Energy* 2011;36(10):5899–908.
- [18] Cheng CH, Yu YJ. Dynamic simulation of a beta-type stirling engine with cam-drive mechanism via the combination of the thermodynamic and dynamic models. *Renewable Energy* 2011;36(2):714–25.
- [19] Cheng C-H, Yu Y-J. Combining dynamic and thermodynamic models for dynamic simulation of a beta-type stirling engine with rhombic-drive mechanism. *Renewable Energy* 2011;37(1):161–73.
- [20] Cinar C, Karabulut H. Manufacturing and testing of a gamma type stirling engine. *Renewable Energy* 2005;30(1):57–66.
- [21] Karabulut H, Cinar C, Aksoy F, Yucesu HS. Improved stirling engine performance through displacer surface treatment. *International Journal of Energy Research* 2010;34(3):275–83.
- [22] Karabulut H, Cinar C, Ozturk E, Yucesu HS. Torque and power characteristics of a helium charged stirling engine with a lever controlled displacer driving mechanism. *Renewable Energy* 2010;35(1):138–43.
- [23] Sripakagorn A, Srikam C. Design and performance of a moderate temperature difference stirling engine. *Renewable Energy* 2011;36(6):1728–33.
- [24] Organ AJ. The regenerator and the stirling engine. UK: Mechanical Engineering Publications; 1997. p. 34–7.
- [25] Cheng CH, Yang HS. Optimization of geometrical parameters for stirling engines based on theoretical analysis. *Applied Energy* 2012;92:395–405.
- [26] Kirkley DW. Determination of the optimum configuration for a stirling engine. *Journal of Mechanical Engineering Science* 1962;4(3):204–12.
- [27] Organ AJ. Thermodynamics and gas dynamics of the stirling cycle machine. Cambridge: Cambridge University Press; 1992. p. 310–4.
- [28] Moffat RJ. Describing the uncertainties in experimental results. *Experimental Thermal and Fluid Science* 1988;1(1):3–17.
- [29] El-Ehwany AA, Hennes GM, Eid EI, El-Kenany E. Experimental investigation of the performance of an elbow-bend type heat exchanger with a water tube bank used as a heater or cooler in alpha-type stirling machines. *Renewable Energy* 2011;36(2):488–97.
- [30] Tanaka M, Yamashita I, Chisaka F. Flow and heat transfer characteristics of the stirling engine regenerator in an oscillating flow. *JSME international journal Ser 2, Fluids engineering, Heat Transfer, Power, Combustion, Thermophysical Properties* 1990;33(2):283–9.
- [31] Eid E. Performance of a beta-configuration heat engine having a regenerative displacer. *Renewable Energy* 2009;34(11):2404–13.